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FILM BOILING AND FREE CONVECTION HEAT TRANSFER TO CARBON DIOXIDE NEAR THE CRITICAL STATE

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Abstract—Measurements of the heat transfer from a horizontal platinum wire to carbon dioxide over a pressure range from 0·8 to 1·1 of the critical pressure are reported. The heat transfer coefficient at conditions very close to the critical point is very large. Below the critical point in the two-phase region, film boiling appears with uprising vapor in columns or sheets. The sheets are similar in appearance to free convection flows above the critical point. Two similar relations correlate the heat-transfer measurements at bulk conditions both above and below the critical point.

NOMENCLATURE

A_2, A_3, A_4, A_5, constants from equation (A.1);
a, Laplace constant $= (\sigma/g\Delta\rho)^{\frac{1}{2}}$ [mm];
C_2, C_3, C_4, constants from equation (A.1);
C_5, C_6, constants in equation (A.2);
C_7, C_8, constants in equation (A.3);
c_p, specific heat at constant pressure;
D, wire diameter;
g, gravitational acceleration;
h, heat-transfer coefficient $= q/\Delta t$ [W/m^2 °C];
i, enthalpy;
k, thermal conductivity;
$\check{K}$, constant in equation (A.1);
K, constant in Ra^* and Nu^*—see text;
L, characteristic length;
m, exponent from (A.3);
n, exponent from (A.3);
Nu, Nusselt number $= qD/k\,\Delta t$;
Nu^*, modified Nusselt number $= (qD/k\,\Delta t)\,K$;
P, pressure [bar];
r, exponent in equation (A.2);
q, heat flux [W/m^2];
q_{max}, heat flux at boiling crisis;
q_{min}, minimum heat flux point on boiling curve;
Ra, Rayleigh number

$$= Gr \times Pr = \frac{D^3\rho(\rho_b - \rho_w)\,c_p g}{\mu k};$$

Ra^*, modified Rayleigh number

$$= \frac{D^3\rho(\rho_b - \rho_w)\,(i_w - i_s)g}{\mu k \quad \Delta t} K;$$

s, exponent from equation (A.2);
T, absolute temperature;
t, temperature [°C];
Δt, temperature difference $= t_w - t_b$ [°C];
Δt_{min}, temperature difference at q_{min};
v, specific volume.

Greek letters

λ, dominant Taylor wavelength, measured distance between rising vapor columns in film boiling;
μ, viscosity;
ρ, density;

$\Delta\rho$, density difference between liquid and vapor;
σ, surface tension.

Subscripts
b, bulk conditions;
c, critical conditions, (for CO_2, p_c = 73·8 bar and t_c = 31·1°C;
r, reduced value, referred to critical point;
s, saturation conditions;
w, wall conditions.

Non-subscripted properties for Nu and Ra numbers are evaluated $t_f = t_b + 0{\cdot}5\,\Delta t$.

INTRODUCTION

THE HEAT transfer to a fluid which is in a state near its thermodynamic critical point is of considerable interest. A direct application can be made in the heat exchangers and power plants that are operating or will operate in this region. In addition, near the critical point, slight changes of temperature and pressure permit a variety of different types of convective flow and heat transfer to occur. This is due to the very large changes of properties such as surface tension, density, specific heat, thermal conductivity, etc. Nucleate boiling, film boiling, and convection with large property variations are possible with only slight changes of the thermodynamic state of the fluid causing a given system to change from one mode of heat transfer to another.

The present study follows two previous investigations of heat transfer in the critical region. The first study was of free convection from a wire to carbon dioxide in the supercritical region [1] while the second [2–4] was of boiling heat transfer to carbon dioxide in the subcritical region.

Goldstein and Aung [1] indicate that the usual free-convection correlations for heat transfer can be used near the critical state. Contrary to investigations by Nishikawa and Miyabe [5] and Knapp and Sabersky [6], they find neither a sharp rise of the heat flux nor a sharp change in the slope of the heat transfer curve in the critical region as the temperature difference is increased Grigull and Abadzic [2–4] demonstrate that, for film boiling in a region close to the critical, the heat transfer coefficient for any particular heat flux is independent of pressure.

The present investigation includes measurements in both the subcritical and supercritical regions of carbon dioxide. The range of pressure is from about 0·8 of the critical pressure to approximately 1·1 times the critical pressure. During the boiling runs the pressure is subcritical and the fluid temperature is always at the saturation temperature of carbon dioxide. Above the critical point the fluid temperature is varied to obtain large variations in the bulk density.

During the course of the study some apparently anomolous density distributions were observed, when the fluid was close to and slightly above the critical point. Some time after the heating of the wire had terminated when calculations indicated that the temperature distribution should be uniform a significant density gradient in the vertical direction was found. A simple optical system used to study this phenomenon and possible explanations are described in Appendix B.

EXPERIMENTAL APPARATUS

The experimental apparatus used for this investigation is basically the same as that described in [1]. An electrically heated platinum wire is mounted horizontally in a cylindrical pressure chamber filled with carbon dioxide. Pressure measurements, made with Bourdon and dead weight gauges, and temperature measurements from thermocouples, indicate the bulk conditions in the chamber. The heat flux and temperature of the platinum wire are calculated from the current and voltage drop across a 100 mm section of the wire. The pressure chamber is provided with viewing windows. The original apparatus was modified to enhance observation of the test wire. Viewing tubes pass through the water immersion tank (used to maintain the high pressure chamber at constant temperature) and connect to the pressure chamber on both sides, eliminating the previously encountered distortion due to the water layer.

Two different platinum wire diameters are used. The smaller wire ($D = 0{\cdot}076$ mm) is used primarily when the pressure is above critical. In an earlier investigation made in this region [1] a 0·38 mm dia. wire was used, but the higher heat input from the large wire causes difficulties in maintaining constant pressure and temperature. No problems of this nature are encountered with the smaller wire of this investigation. A larger wire ($D = 0{\cdot}38$ mm) is used in the two-phase, subcritical region.

To reduce the influence of impurities, the pressure chamber and wire are cleaned with solvents. For the first runs, commercially pure carbon dioxide was introduced into the chamber as a vapor and condensed. However, after some time, impurities, believed to be oil particles, were deposited upon the test wire. These deposits appear to influence the fluid circulation near the wire, and the heat transfer measurements are then not reproducible. Since these preliminary tests a high purity carbon dioxide supply has been used. Visual observation indicate that the impurity deposits on the wire are eliminated and the heat-transfer measurements are then reproducible throughout the investigation.

DISCUSSION OF RESULTS

A. *Boiling in the subcritical region*

Moderate heat transfer from a heating surface to a saturated liquid usually results in nucleate or bubble boiling. A change to film boiling occurs with an increase of the heat flux over a certain q_{max}. Previous investigations of boiling heat transfer by Grigull and Abadzic [3, 4] show that the heat-transfer coefficient for nucleate boiling rapidly increases with an increase in pressure, and becomes very large as the critical point is approached. At the same time the heat flux for boiling crisis, q_{max}, decreases towards zero at pressures close to the critical. Thus, the large nucleate boiling heat-transfer coefficient has little practical significance.

At conditions far below critical, the temperature difference at which the minimum film boiling heat flux occurs, Δt_{min}, has an order of magnitude of several hundred degrees. In the critical region, q_{min} and Δt_{min} rapidly approach zero as the surface tension decreases to zero at the critical point.

The influence of surface tension on film boiling heat transfer as the critical point is approached would be expected to decrease, and the heat transfer by film boiling in this region should become more characteristic of single-phase free convection. The visual observations and the correlation of measurements of film boiling phenomena in the critical region support this viewpoint.

Photographs of film boiling presented in Figs. 1–5 show significant changes of the flow patterns in the critical region. At a pressure of about 69·5 bar, individual vapor bubbles rise from the film blanket surrounding the wire at nearly regular spacing (Figs. 1 and 2). With increasing heat flux at constant pressure the horizontal distance between the rising bubbles remains almost constant and the bubble frequency increases. With an increase in pressure, the spacing between the rising bubbles decreases.

At pressures very close to the critical, individual vapor bubbles unite to form vapor columns (Fig. 3). As the heat flux increases, the vapor columns join together (Fig. 4) and sheets of vapor rise from the wire (Fig. 5).

Vapor sheets appeared in a previous investigation [2] at about 1°C below the critical with a wire which was approximately one-fourth the diameter of that used in the present study. With the larger wire, the vapor sheets begin to appear at a bulk temperature about 4°C below the critical. Oscillations in the laminar flow (see Fig. 5) are also observed. As discussed below, the horizontal spacing of the columns and bubbles

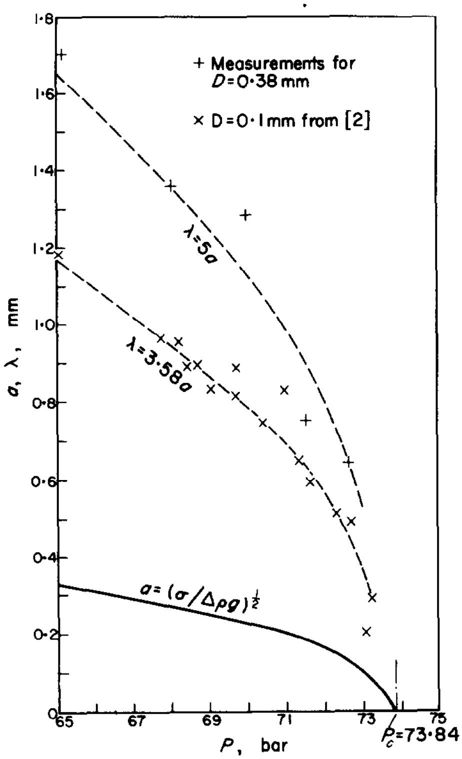

FIG. 6. Laplace constant for CO_2 and dominant wavelength vs. pressure.

for film boiling increase with wire diameter.

With a wire diameter of 0·38 mm as used in the present investigation, the ratio of the measured horizontal spacing between the bubbles or columns, λ (dominant wavelength), to the Laplace constant, a, is found to be approximately 5. The Laplace constant, a, is given by

$$a = (\sigma/g\Delta\rho)^{\frac{1}{2}}.$$

For a wire of 0·1 mm dia., as shown in [2], $\lambda/a \approx 3{\cdot}5$.

Figure 6 shows the relationship between a and λ in the measured region including the results of Grigull and Abadzic [2] with a 0·1 mm wire. The influence of the wire diameter on λ can be observed.

A constant value of λ/a is not in agreement with the expression derived by Lienhard and Wong [7]. For film boiling on the wires they find,

$$\lambda = \frac{2(\sqrt{3})\,\pi}{\sqrt{[1/(a)^2 + 1/(2L)^2]}}$$

where L is the characteristic length (radius). Although this expression does not predict a constant value of λ/a, as measured, it is in approximate agreement with the values of λ and the effect of wire diameter on λ found in the present measurements. It should be remarked that the observed column spacing, λ, is relatively irregular for the larger (0·38 mm) wire but quite uniform for the smaller (0·1 mm) wire.

The variation of the film boiling heat flux with temperature difference between the wire and liquid is presented in Fig. 7. The variation of heat transfer coefficient for the same measurements is presented in Fig. 8. From Fig. 7 one can conclude that with temperature differences sufficiently greater than Δt_{min}:

1. The heat transfer for any particular Δt generally does not depend upon pressure and
2. The wire diameter has a large influence on the heat transfer.

Thus the governing forces for the heat transfer in this region appear to be similar to those for free convection. No influence of the measured dominant Taylor wave length on the heat transfer is found over a wide range of values. As with free convection, the heat transfer appears to depend only upon the density difference and the characteristic properties of the boundary layer.

At pressures near the critical, the heat transfer coefficient increases rapidly as the temperature difference is made quite small (Fig. 8). The influence of the fluid properties upon the heat transfer is evident.

B. *Free convection in the supercritical region*

The supercritical region of interest for carbon dioxide is essentially bounded by the 30°C and 35°C isotherms with fluid density between 0·2

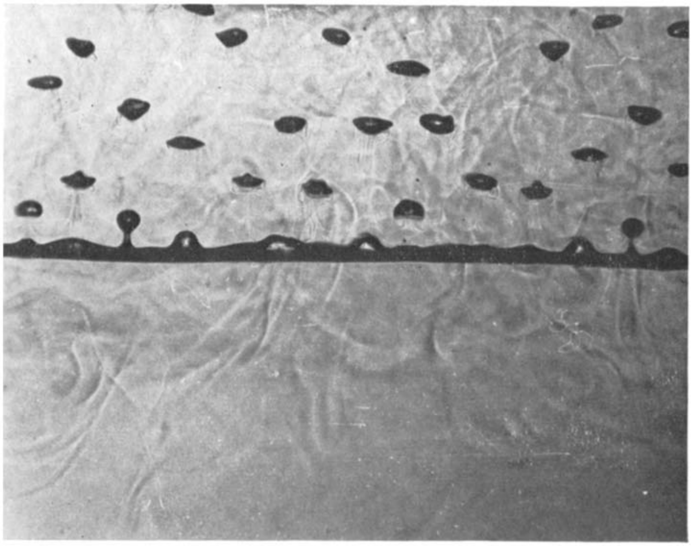

FIG. 1. Film boiling on a wire, $D=0{\cdot}38$ mm. Bulk conditions away from critical point ($t'_s = 27{\cdot}5°C$, $p'_s = 69{\cdot}5$ bar, $q=8 \times 10^4$ w/m^2, $\Delta t = 80{\cdot}0°C$). The wavelength of the vapor film above the wire, $\lambda \simeq 1{\cdot}3$ mm.

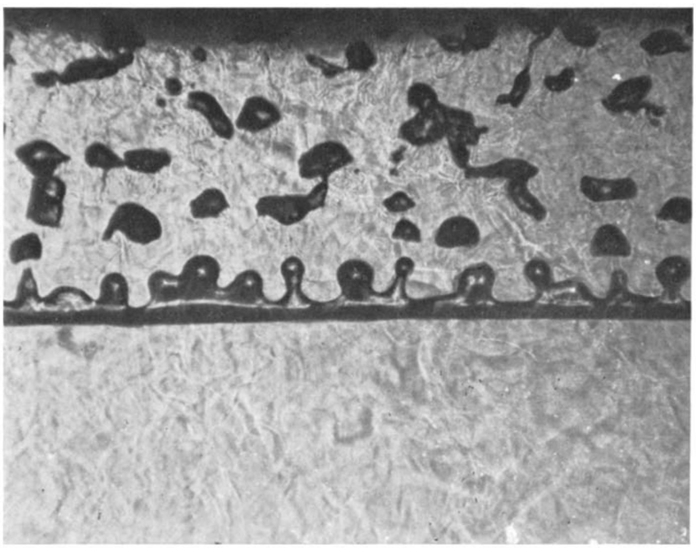

FIG. 2. Film boiling on a wire, $D=0{\cdot}38$ mm. Bulk conditions the same as for Fig. 1 but the heat flux is higher ($q = 4{\cdot}0 \times 10^4$ w/m^2$\Delta t=550°C$). The wavelength $\lambda \simeq 1{\cdot}3$ mm, more waves produce vapor bubbles simultaneously.

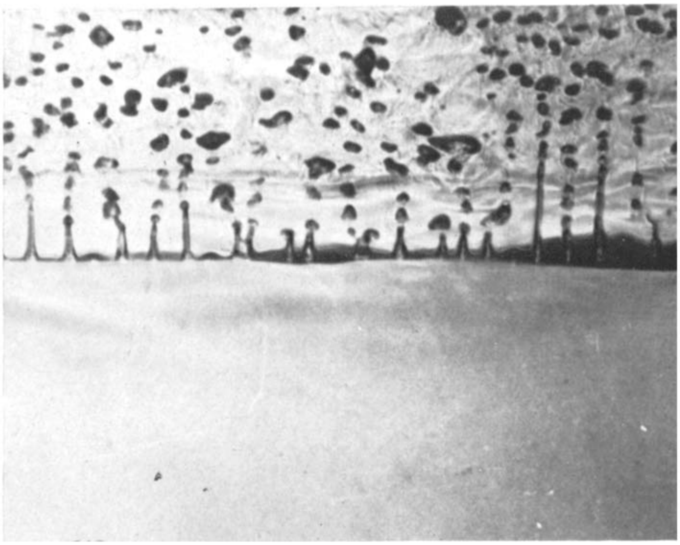

FIG. 3. Film boiling wire, $D = 0{\cdot}38$ mm. Bulk conditions close to critical, formation of vapor columns. Wavelength $\lambda \simeq 0{\cdot}75$ mm. ($t_s = 29{\cdot}5°C$, $p_s = 71{\cdot}4$ bar, $q = 10 \times 10^4 w/m^2$, $\Delta t = 110°C$). (The horizontal lines originate from the light displacement caused by density stratifications.)

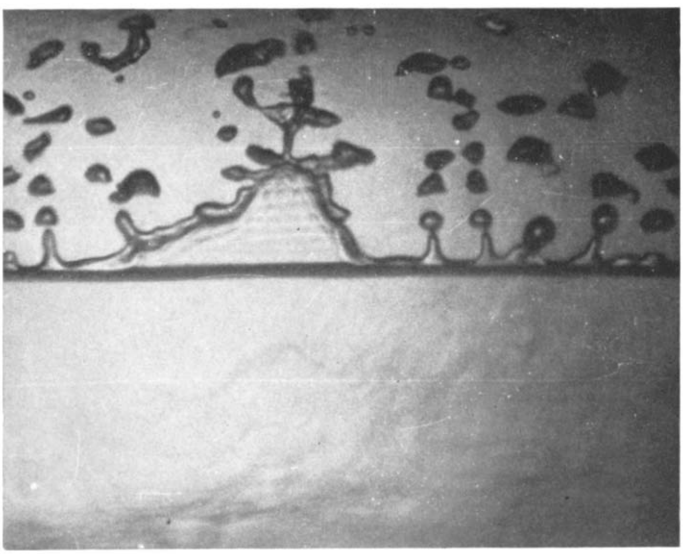

FIG. 4. Film boiling on a wire, $D = 0{\cdot}38$ mm. Bulk conditions the same as for Fig. 3—increased heat flux ($q = 18 \times 10^4$ w/m^2, $\Delta t = 200°C$). The vapor sheet rises from the vapor film.

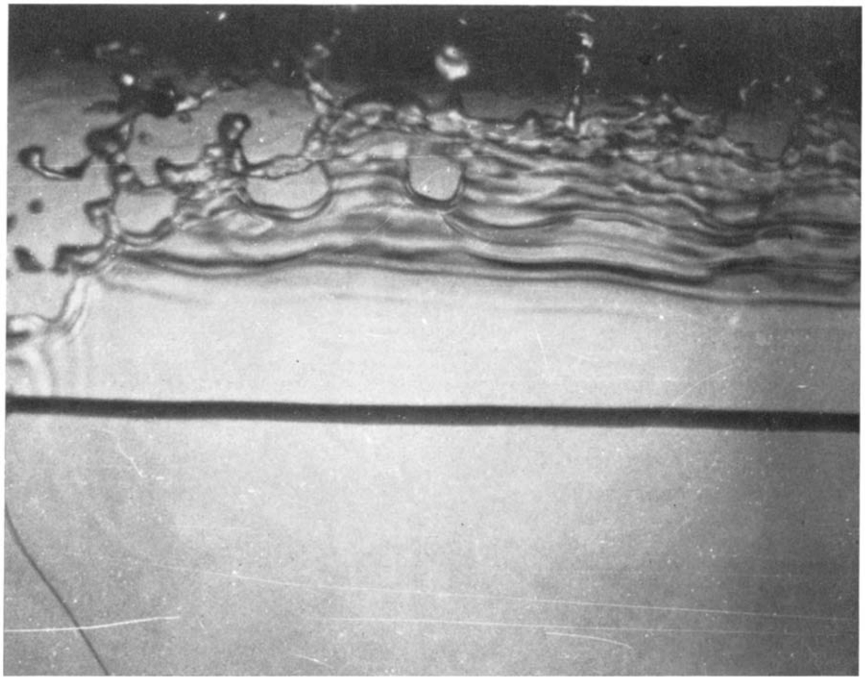

FIG. 5. Film boiling on a wire, $D = 0{\cdot}38$ mm. Bulk conditions the same as for Figs. 3 and 4—heat flux further increased ($q = 35 \times 10^4$ w/m^2, $\Delta t = 400$°C). Vapor sheets over entire length of the wire. Some wavering of the sheet in upper position is evident.

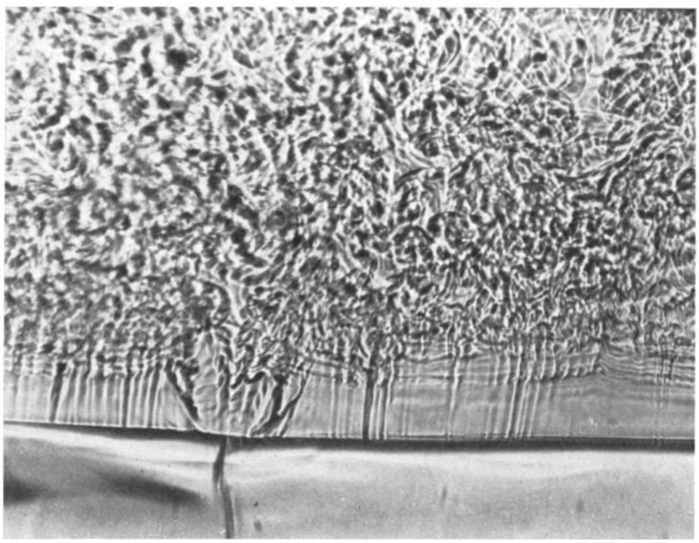

FIG. 9. Free convection from a wire, $D = 0{\cdot}076$ mm. Bulk conditions close to critical point ($t_b = 32{\cdot}0$°C, $p_b = 75$ bar, $q = 5 \times 10^4$ w/m^2, $\Delta t = 22$°C). The similarity to vapor sheets in film boiling is evident.

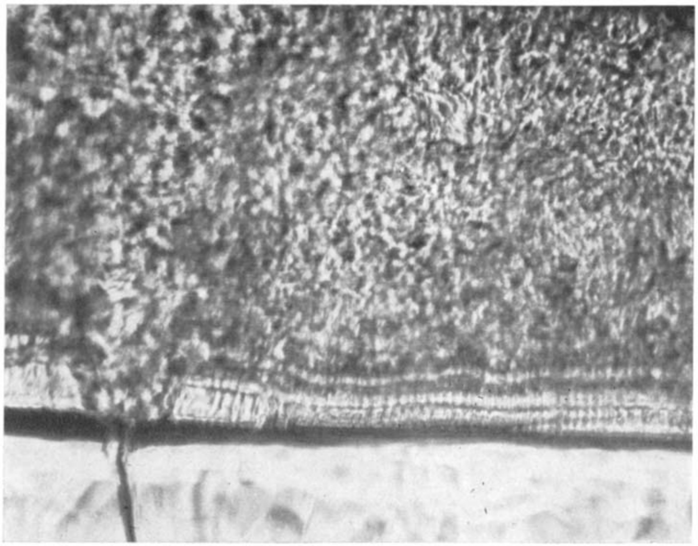

FIG. 10. Free convection from a wire, $D = 0{\cdot}076$ mm. Bulk conditions same as for Fig. 9—increases heat flux ($q = 20 \times 10^4$ w/m^2, $\Delta t = 140°$C).

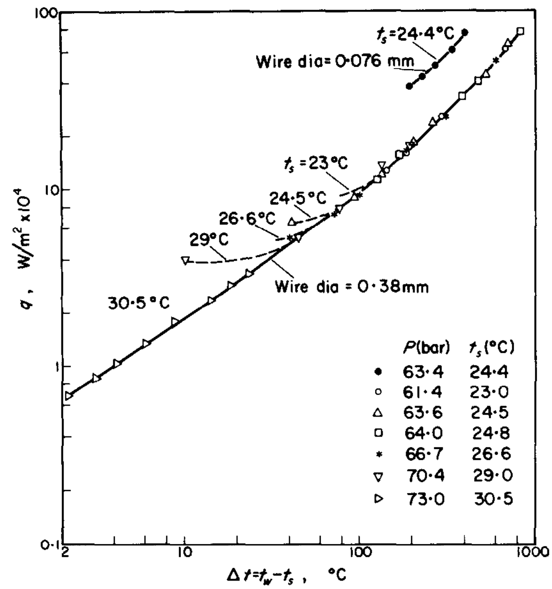

FIG. 7. Heat flux vs. temperature difference for film boiling.

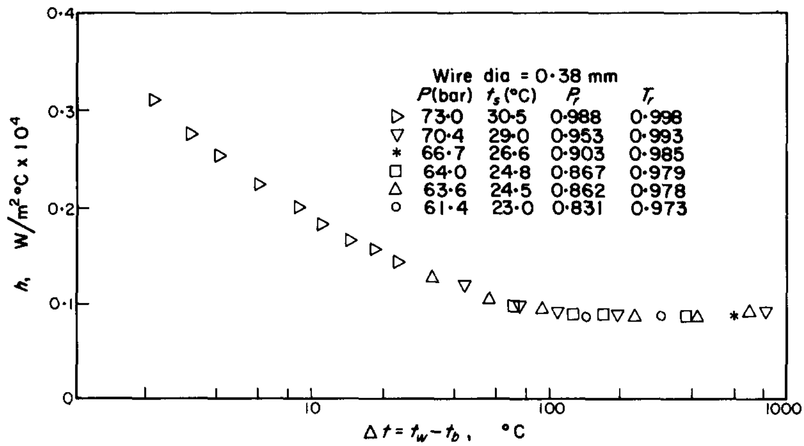

FIG. 8. Heat-transfer coeffcient vs. temperature difference for film boiling.

and 0·8 g/cm^3. The transposed critical temperature at a given pressure is customarily defined as the temperature along the isobar in the supercritical region where the specific heat, c_p, is a maximum. The transposed critical temperature will occur within the thermal boundary layer if the bulk temperature is below and the heating surface temperature is above the transposed critical temperature. Because of the large variation of density (with temperature) at the transposed critical temperature, the boundary layer velocity profile would then differ from that found with free convection far from the critical state.

Observations of free convection in this investigation show a change in flow conditions near the wire similar to that already reported [1]. At low heat flux, laminar flow in the whole field above the wire is observed. With an increase of heat flux, turbulent bursts in the upper region appear. The distances of these turbulent bursts from the wire decreases with further increase of the heat flux (Figs. 9 and 10). The appearance of the turbulent flow occurs at smaller heat fluxes as the bulk conditions approach the critical state. The turbulent bursts induce oscillations in the laminar flow. The oscillations are similar to those observed with film boiling in the subcritical region. At conditions very close to the critical state, the turbulent flow is observed immediately above the wire and visual observations are obscured by the high density and refraction gradients in the fluid. However, no appreciable change in the heat-transfer coefficient variation is found under those conditions. Therefore, it might be concluded that the boundary layer remains laminar.

The free convection heat flux variation with temperature difference between the wire and fluid in the supercritical region is shown in Fig. 11. It might be helpful to mention here that the bulk conditions for all of the tests plotted are shown on a state diagram for carbon dioxide included with the appendix (Fig. 15).† Figure 12 shows the behavior of the heat-transfer coefficient for the same measurements. The heat-transfer coefficient increases rapidly for bulk conditions close to the critical as the temperature difference decreases. Near the critical point, the hysteresis noted in [5], is not observed while either increasing or decreasing the heat flux.

CORRELATION OF RESULTS

One of the earliest correlations of film boiling heat transfer was made for horizontal tubes by Bromley [8], using laminar film theory. This was modified for wires and for fluids of low latent heat of vaporization by Frederking [9] and other investigators [10, 11]. Pitschmann [12] has shown that for horizontal wires and circular tubes one empirical relation presents the results of several investigations over a wide range of fluids and bulk conditions. This empirical relation for conditions far away from the critical is

$$Nu^* = 0{\cdot}9\, Ra^{*0{\cdot}08} + 0{\cdot}8\, Ra^{*0{\cdot}2} + 0{\cdot}02\, Ra^{*0{\cdot}4}.$$

The range of Rayleigh numbers in which the above relation fits the experimental data, as given by [12], is Ra^* from 10^{-3} to 10^{10} where modified Nusselt and Rayleigh numbers are given by

$$Nu^* = \frac{qD}{\Delta t\, k} K$$

$$Ra^* = \frac{D^3 \rho(\rho_b - \rho_w)\, g(i_w - i_s)}{k\mu\Delta t} K.$$

The constant K in this equation varies slightly from unity only at higher temperature differences (influence of radiation) and at very low pressure or with very small wires (due to accomodation effects). The slope of the curve presented by this equation is similar to the laminar free convection curve of McAdams [13]. The Nusselt numbers according to this correlation for film boiling are roughly 30 per cent higher than for free convection at the equivalent Rayleigh numbers.

The visual observations made in this study shows the similarity between film boiling in the subcritical and free convection in the supercritical regions. The similarity of the vapor sheets in film boiling to the laminar flow above the wire during free convection is quite evident. The turbulent bursts above the sheets or laminar flow induce characteristic oscillations in both cases. Similarity in flow structure is also evident in the form of vertical stripes (apparently representing wave crests and troughs) on the sheets. Therefore, it may be interesting to compare the results of the heat-transfer measurements in those regions and the correlations by Pitschmann and McAdams.

† Note that one of the runs (p = 52·2 bar) is far below the critical state.

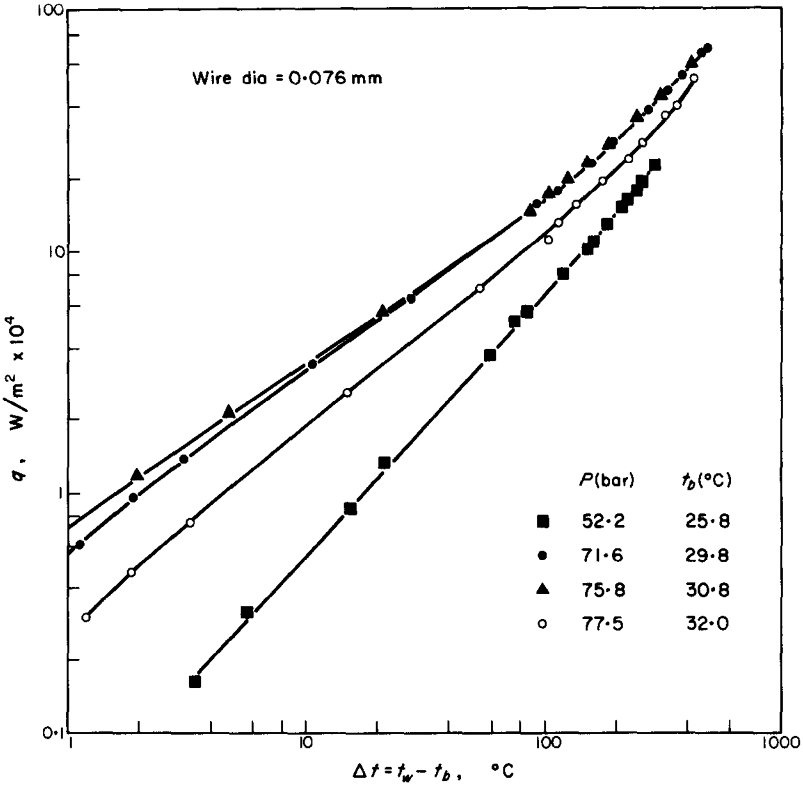

FIG. 11. Heat flux vs. temperature difference for free convection.

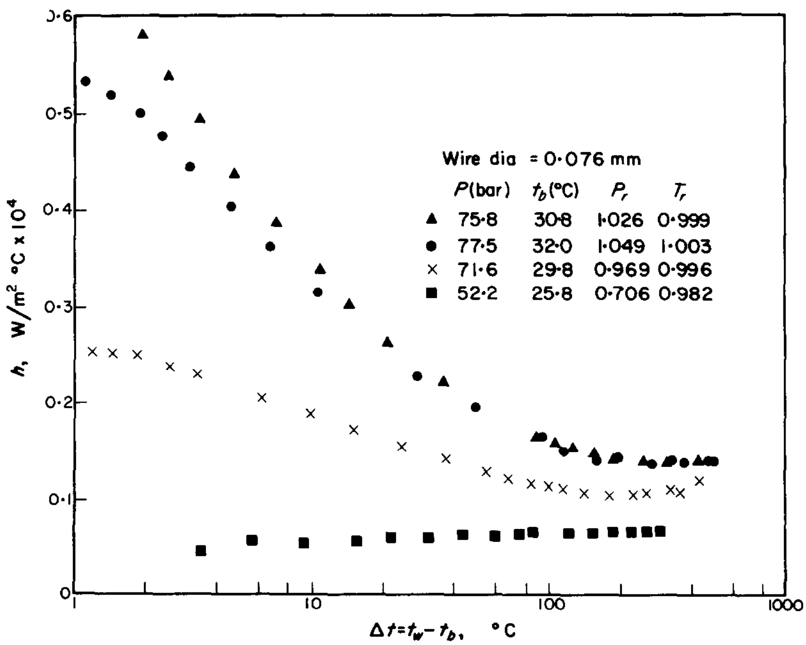

FIG. 12. Heat-transfer coefficient vs. temperature difference for free convection.

To compare the results of the present measurements with previous studies a Rayleigh number based on density difference is used,

$$Ra = \frac{D^3 \rho(\rho_b - \rho_w) c_p g}{\mu k}.$$

Since the latent heat of vaporization near the critical point is small the difference between Ra and Ra^* in this region is small. The Nusselt number is defined by

$$Nu = \frac{qD}{\Delta t k}.$$

The properties without subscripts are evaluated at the film temperature, t_f, which is the average of the wall and bulk fluid temperatures. Values of heat flux, q, and temperature difference, Δt, were obtained for correlation purposes from the curves in Figs. 7 and 11. For convenience points were taken from the smoothed curve of q vs. Δt at a given pressure. Significant errors are possible in the estimation of fluid properties, especially in the values of c_p and k, at states close to the critical. Therefore, for pressures close to the critical, the minimum temperature differences used for evaluation of Nu and Ra numbers are 10°C for free convection and 3·6°C for film boiling. The trend of the present results does not agree with the predictions of Breen and Westwater [11] and Baumeister and Hamill [14]. Their predictions indicate a significant influence of the surface tension on film boiling heat transfer. That this does not occur may be concluded from Fig. 7.

A comparison of the representative test results, and the correlations for film boiling [12] and free convection [13], is shown in Fig. 13. Solid symbols represent the measurements for free convection and open symbols represent the same measurements for film boiling in the subcritical region.

From Fig. 13 the following conclusion can be drawn concerning film boiling:

1. Film boiling heat transfer in the critical region can be described by one correlation based on the Nusselt and Rayleigh numbers.
2. The high rate of change of the dominant Taylor wavelength (shown in Fig. 6) and different flow patterns of uprising vapor in the observed region do not significantly influence the heat transfer.
3. Very close to the critical pressure (at $P = 73$ bar) a sharp departure of the measurements from the correlation occurs as the temperature difference decreased (Ra increased). This result can not be explained without a better knowledge of the thermal conductivity in the critical region.

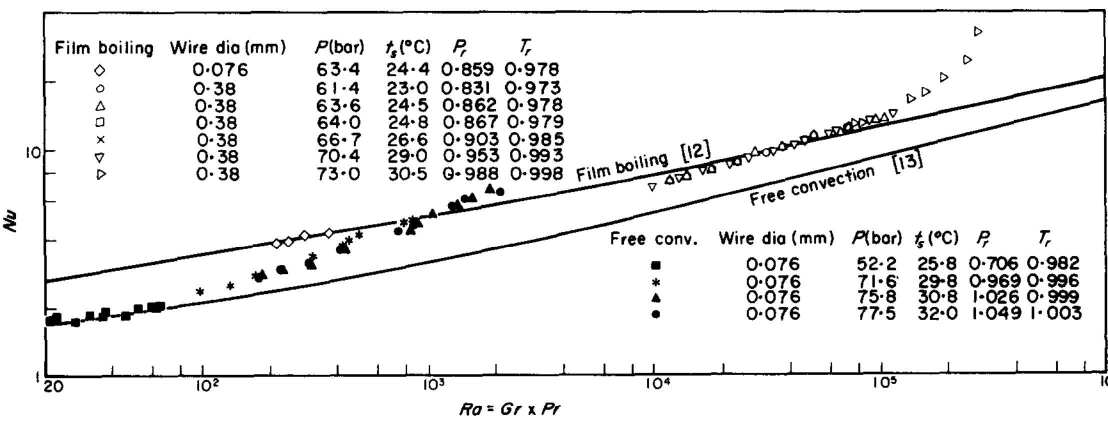

FIG. 13. Nusselt number vs. Rayleigh number—comparison of film boiling and free convection.

The free convection data plotted on the same figure lead one to conclude:

1. The heat-transfer measurements for free convection of carbon dioxide on a wire at conditions away from the critical point ($P = 52{\cdot}2$ bar) are well represented by a standard free convection correlation [13].
2. The results in the supercritical region generally indicate higher Nusselt numbers than those given by the correlation [13]. The largest differences occur at pressures close to the critical and when the temperature difference is small. These higher Nusselt numbers can possibly be explained by the existence of a pseudo-film-boiling effect in this region; the high change of density near the transposed critical temperature in the laminar boundary layer causes a film-boiling-like velocity distribution.

Using the same procedure, the measured data for free convection in the supercritical region by Goldstein and Aung [1] and Nishikawa and Miyabe [5] are compared with the same correlations and presented in Fig. 14.

The results of this comparison also show a higher Nusselt number than predicted by the free convection correlation. The results of the heat-transfer measurements from [1] are close to those of the present study. The data from Nishikawa and Miyabe [5] fall somewhat higher on the figure.

SUMMARY

The heat-transfer coefficient for a small imposed temperature difference shows a sharp increase at bulk conditions approaching the critical. This behavior is evident in both the subcritical and supercritical regions. For the same bulk conditions, the heat-transfer coefficient is reduced appreciably at larger temperature differences.

Heat transfer in the two-phase region well below the critical point can result in either nucleate or film boiling. In the critical region, film boiling will be established and maintained even at small heat fluxes and temperature

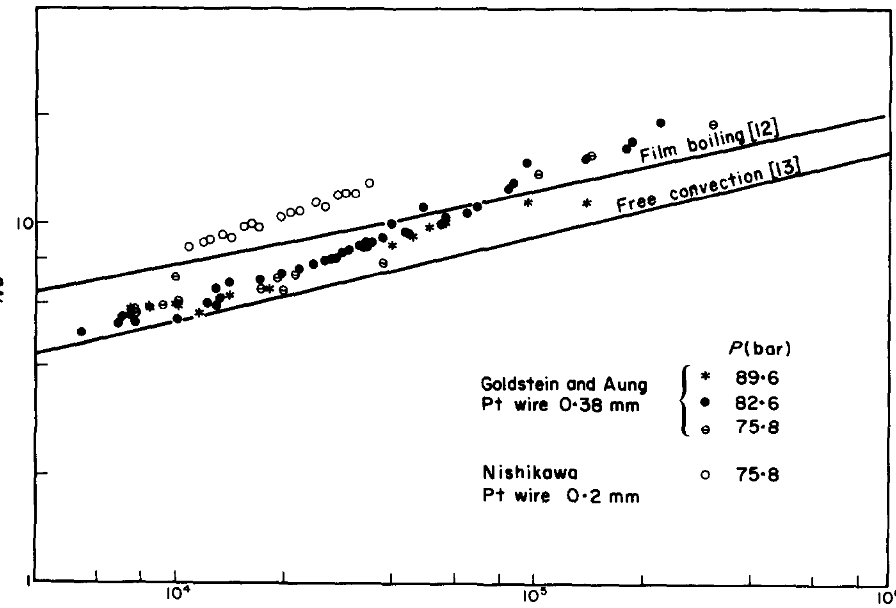

FIG. 14. Nusselt number vs. Rayleigh number—from previous studies.

differences. Film boiling in the critical region results in various characteristic shapes of the uprising vapor; bubbles, columns and sheets. The characteristic Taylor wave length is strongly dependent on the pressure and wire diameter for film boiling in the critical region. At bulk conditions very close to the critical point, the vapor rises in the form of sheets at all temperature differences. Neither the vapor configuration nor the variation of the characteristic Taylor wave length has any appreciable influence upon the heat transfer for film boiling. Above the critical pressure no sharp discontinuities in the heat flux vs. temperature curve are observed.

It appears that simple relations can correlate heat transfer measurements for film boiling in the subcritical region and free convection in the supercritical region. These relations give satisfactory results even for pressures close to the critical provided the temperature difference between the wire and the fluid is not too small. At small temperature differences and conditions very close to the critical the relations predict lower values of Nusselt number than indicated by measurement.

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APPENDIX A

Thermodynamic and Transport Properties of CO_2

There is relatively sparse information available on the thermophysical properties of most substances in the region of their thermodynamic critical states. In this region the properties of carbon dioxide are probably better known than those of any other substance. This, along with the relative ease of attaining its critical temperature and pressure are the major reasons for using carbon dioxide in many studies near the critical state. Even so, the behavior of the thermal conductivity of carbon dioxide is not well known (see Simon and Eckert [15]). Additionally, Straub, [16], has shown that density stratification in a fluid close to the critical point under the influence of a gravity field prevents consideration of the system as a truly homogeneous one.

Density

To estimate density in the supercritical and subcritical vapor region for the correlations of heat transfer measurements, an empirical p–V–T equation, given by Martin [17], is used. This equation is especially designed to have correct curvature in the neighborhood of the critical point and is given as

$$p = \frac{RT}{v-b} + \frac{A_2 + B_2 T + C_2 \exp(-\bar{k}T)}{(v-b)^2} + \frac{A_3 + B_3 T + C_3 \exp(-\bar{k}T)}{(v-b)^3} + \frac{A_4}{(v-b)^4} + \frac{A_5 + B_5 T + C_4 \exp(-\bar{k}T)}{(v-b)^5}. \qquad (A.1)$$

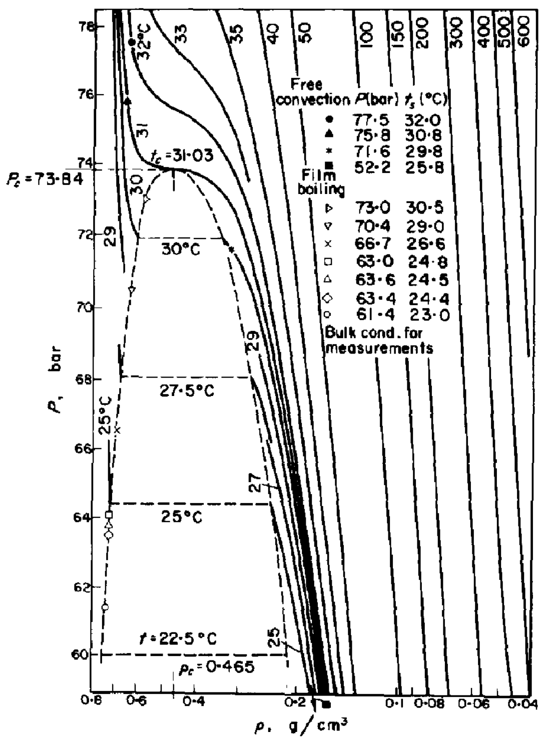

FIG. 15. Pressure vs. density, state diagram for CO_2.

Where p is in psia, T in °R and v in ft³/lb. The constants used for carbon dioxide are given in Table 1 from [17].

Figure 15 shows a pressure–density diagram constructed using this equation. The comparison of values from this equation with measured isotherms of Straub [16] and Michels *et al.* [18], gives good agreement except for the region within 1°C of critical. Also included in Fig. 15 are the symbols representing the bulk conditions for the heat transfer tests. Computation of desired densities from equation (A.1) is done by iteration. The density of saturated liquid, for correlations in the two phase region, are estimated from tabulated data by Planck [19].

Thermal conductivity

The thermal conductivity of carbon dioxide is calculated from a reduced state correlation for carbon dioxide presented graphically by Kennedy and Thodos [21]. This correlation is based on several different sets of measurements made in a temperature range $T_r = 0{\cdot}6$–$T_r = 3{\cdot}5$ and extrapolated to $T_r = 10$. For easier computation, this correlation is approximated by the equation

$$k = \{C_5(T_r)^s + C_6(\rho_r)^r\} \times 10^{-5}\ [\text{cal/s cm °K}]. \qquad (A.2)$$

The constants are given as

Reduced temp. range (T_r)	C_5	s
0·5–1·5	4·03	1·19
1·5–4·0	4·265	1·05
4·0–10·0	5·607	0·85
Reduced density range ρ_r	C_6	r
0·05–0·2	6·0	1·17
0·2–0·5	7·396	1·30
0·5–3·0	7·763	1·37

Viscosity

The viscosity of CO_2 is obtained from another reduced state correlation. This correlation [21] is also given graphically and for easier computation, is approximated by the equation

$$\mu = \{C_7(T_r)^n + C_8(\rho_r)^m\} \times 10^{-5}\ [\text{cp}]. \qquad (A.3)$$

The constants used are

Reduced temp. range (T_r)	C_7	n
0·4–1·0	1520	0·96
1·0–2·0	1520	0·85
2·0–20·0	1724	0·668
Reduced density range (ρ_r)	C_8	m
0·1–1·0	1815	1·57
1·0–2·0	1815	2·15
2·0–4·0	503·4	4·00

Specific heat

The specific heat of CO_2 is determined from a graphical presentation of data from Michels and De Groot [20] and the National Bureau of Standards, Table [22].

Table 1. *Constants for the equation* (A.1) from [17] (*Units are consistent with* p *in psia*, Γ *in* °*R and* v *in ft*^{3}/*lb*)

$R = 0{\cdot}24381$	$C_3 = 4{\cdot}705805$
$b = 0{\cdot}007495$	$A_4 = -2{\cdot}112459 \times 10^{-3}$
$A_2 = -8{\cdot}9273631$	$A_5 = 7{\cdot}017835 \times 10^{-6}$
$B_2 = 5{\cdot}262476 \times 10^{-3}$	$B_5 = 1{\cdot}023511 \times 10^{-8}$
$C_2 = -150{\cdot}97587$	$C_4 = 4{\cdot}55437 \times 10^{-4}$
$A_3 = 0{\cdot}17948621$	$k = 0{\cdot}01$
$B_3 = -5{\cdot}770542 \times 10^{-5}$	

APPENDIX B

The optical technique used to study the density distribution in the high pressure chamber makes use of a deflection mapping system [23]. Light from a laser passes through a beam expander to produce a parallel beam. This beam passes through a slit inclined at an angle approximately 45° to the vertical, then through the high pressure carbon dioxide chamber and on to a screen. If the index of refraction does not vary normal to the light beam the image on the screen is an inclined straight line as indicated by "1" in Fig. 16. If, however, the index of refraction varies, for example, in the vertical direction, the straight line will be distorted as indicated by the other curves in Fig. 16.

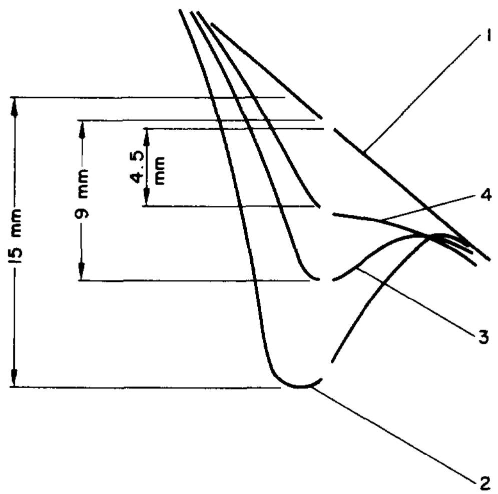

FIG. 16. Image of the slit on the screen in the deflection mapping study. The interruption of the lines is due to the shadow of the heating wire. Table 2 lists the conditions under which each curve was obtained.

The displacement of a light ray on the screen, from the position when the field is uniform, is proportional to the gradient of the index of refraction in the vertical direction. If the Lorenz–Lorentz relationship for the variation of the index of refraction holds, the density gradient in the vertical direction is found to be closely proportional to the displacement of the light beam due to this density variation.

Typical test results are shown in Fig. 16, which was taken when the test section contained carbon dioxide initially at a uniform temperature of 31·0°C and pressure of 74·14 bar, corresponding to a state on the pressure density diagram about 0·6 bar above the co-existence line and with a mean density of 0·58 g/cm^3. The straight line indicated by "1" was obtained with an initially uniform density distribution over a height of about 3 cm in the middle of the test chamber. The gap in the line indicates the position of the heating wire. The wire then was heated with a heat flux of about 10^5 W/m^2 for about 80 s, the total heat input during this time being only 3·2 W·s. Thus neither the pressure nor mean temperature were significantly affected. During the heating, fluid beneath the wire was not involved in convection and a shadowgraph observation indicated that a slow motion had disappeared within 10 min after the heating had ceased.

Figure 16 shows the image at the screen at various times after the cessation of heating. The times when the curves were obtained and the maximum density gradient (calculated from the maximum deflection) are shown in Table 2. Note the relatively large gradient present even 100 min after the heating had ceased.

Table 2

Curve number on Fig. 16	Time interval after heating (min)	Vertical density gradient (g/cm^4)
1	Uniform state before heating	
2	20	$5{\cdot}5 \times 10^{-3}$
3	50	$3{\cdot}3 \times 10^{-3}$
4	100	$1{\cdot}6 \times 10^{-3}$

Significant density gradients have been reported near the critical state, but under somewhat different conditions. Thus Straub [16] found density stratification at conditions very close to the critical point after small changes in the mean temperature or pressure of the fluid. The highest density gradient was found in the horizontal plane of the fluid where the meniscus had disappeared when crossing the co-existence line. Timrot and Sujskaja [24] observed significant density stratification of carbon dioxide near the critical point when impurities (~3·5 per cent) were present. The density gradient was considerably higher than in pure CO_2 and the concentration varied significantly over the height of the chamber.

In the present test, however, the density gradient is considerably below the plane of the meniscus. The observed stratification could presumably be attributed, at least in part, to concentration differences of impurities and perhaps also to small transient temperature differences in the fluid. However, it is not clear that these are the only effects. The carbon dioxide comes from cylinders of relatively high purity and care is taken to maintain purity during the tests. A simple calculation of the difussion of heat from the heating wire indicates that over the time period in which there is still considerable density variation the temperature should be very close to uniform. If essentially pure CO_2 is present in the test section and the temperature is closely uniform, the density variation could indicate hysteresis in the equation of state near the critical point. However, more systematic measurements should be made to study the density near the critical point.

EBULLITION PAR FILM ET TRANSPORT DE CHALEUR PAR CONVECTION NATURELLE DANS DU GAZ CARBONIQUE PRÈS DU POINT CRITIQUE

Résumé—On décrit les mesures du transport de chaleur à partir d'un fil de platine horizontal dans du gaz carbonique dans une gamme de pression allant de 0,8 à 1,1 fois la pression critique.

Le coefficient de transport de chaleur dans des conditions très voisines du point critique est très élevé. En dessous du point critique dans la région diphasique, l'ébullition par film apparaît avec de la vapeur montant en colonnes ou en nappes. Les nappes ressemblent aux écoulements de convection naturelle au-dessus du point critique. Deux relations semblables corrèlent les mesures de transport de chaleur dans les conditions globales au-dessus et au-dessous du point critique.

WÄRMEÜBERGANG BEIM FILMSIEDEN UND BEI FREIER KONVEKTION AN KOHLENDIOXID NAHE DEM KRITISCHEN ZUSTAND

Zusammenfassung—Es wird über Messungen des Wärmeüberganges von einem horizontalen Platindraht an Kohlendioxid in einem, auf den kritischen Druck bezogen, Druckbereich von 0,8 bis 1,1 berichtet. Die Wärmeübergangskoeffizienten sehr nahe dem kritischen Punkt sind sehr gross. Im Zweiphasengebiet des unterkritischen Bereiches tritt Filmsieden in Form von aufsteigenden Dampfsäulen oder in Schichten auf. Diese Schichten haben ähnliches Aussehen wie die freien Konvektionsströmungen im unterkritischen Gebiet. Zwei ähnlich aufgebaute Beziehungen korrelieren die Wärmeübergangsmessungen bei Temperaturen über und unter dem kritischen Punkt.

ПЛЕНОЧНОЕ КИПЕНИЕ И ПЕРЕНОС ТЕПЛА К ДВУОКИСИ УГЛЕРОДА ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В СОСТОЯНИИ БЛИЗКОМУ К КРИТИЧЕСКОМУ

Аннотация—Пленочное кипение и перенос тепла к двуокиси углерода при естественной конвекции в состоянии близкому к критическому. Доклад об измерениях переноса тепла от горизонтальной платиновой проволоки к двуокиси углерода при диапазоне критического давления от 0,8 до I, I. Коэффициент теплоотдачи в окрестности близкой к критической точке очень высок. В двухфазной области ниже критической точки, появляется пленочное кипение с паром поднимающимся в виде столбцов или пелен. Надкритической точкой пелены с виду подобны свободным конвекционным течениям. Два подобных соотношения коррелируют измерения переноса тепла общей массой в надкритической и ниже критической точках.